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1.0 MOLECULAR TREATMENT OF PROPERTIES OF MATTER

Matter is normally classified as being in one of 3 states: Solids, liquid or gas. From everyday experience, we know that a solid has a definite volume and shape, Liquid has a definite volume but no definite shape and finally, we know that an unconfined gas has neither a definite volume nor definite shape. These characteristics help as picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids but over long periods of time, they tend to flow like liquids.

Likewise, substances can be a solid, a liquid, or a gas (or a combination of any of these), depending on the temperature and pressure. Therefore, the time it takes a particular substance to change its shape in response to an external force determine whether we treat the substance as a solid, as a liquid or as a gas.

1.1 Elasticity

Elasticity is the ability of a substance to regain its original shape and size after being distorted or change by an external force.

E.g : Suspension of a load from the end of a coil of spring of wire, the spring stretches certain length, when the load is removed the spring returns to his former length.

1.1.1 An Elastic Material

An Elastic Material is the one that regains its original shape and size after the distorting force has been removed. The shape and size of a material can be distorted by either compressing or stretching it. And as soon as the compressing or stretching force is removed, an Elastic Material regains its original size.

1.1.2 Elastic Limit:

Elastic Limit is the border line of force beyond which the stretched wire does not return to its original length when the stretching force is removed. i.e. beyond the elastic limit the material loses its elasticity.

1.1.3 Yield point:

Yield point is the point beyond the elastic limit in which the elastic material has yielded all its elasticity permanently and become plastic.

1.1.4 Breaking point:

Breaking point is the maximum extension reached where the material may finally snap or break entirely.

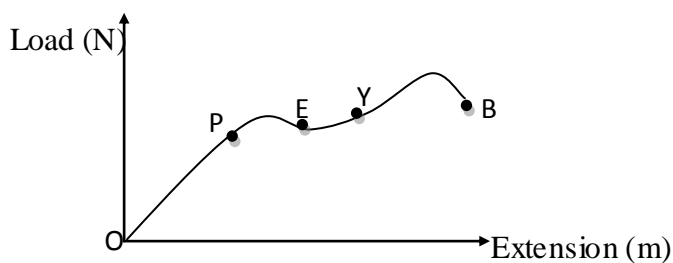


Fig. 1: The graph of extension against load up to breaking point

Where

P = Proportionality limit, E = Elastic limit, Y = Yield point, B = Breaking point, OE = Elastic region, EB = Plastic region

1.1.5 Elastic Constant or Stiffness:

Elastic constant or stiffness of an elastic material is the force required to give a unit extension.

$$k = \frac{F}{e} = \frac{N}{m} = Nm^{-1}$$

1.2 Hooke's Law

Hooke's law is the law that governs the relationship between the stretching force and extension produced in the elastic material. When a load is attached to one end of a fixed string at a certain stage, Hooke's law is obeyed. A point is later reached at which if more load is added, such that the load added is not proportional to the weight of the material, then there will be a breakdown, this point is called the **BREAKING POINT**.

Therefore, Hooke's Law states that provided the elastic limit of an elastic material is not exceeded, the extension (e) of the material is directly proportional to the load or applied force (F).

Mathematically,

$$F \propto e \quad 1$$

$$F = ke \quad 2$$

Where k is a constant of proportionality

1.3 Elastic Properties of Solids

We shall discuss the deformation of solids in terms of the concepts of stress and strain. Let F be the force (N) acting on a body of length (l_0) with an extension (e). If the cross-sectional area of the body is given as (A), then Hooke's law can still be defined as:

$$\frac{F}{A} \propto \frac{e}{l_0} \quad 3$$

1.3.1 STRESS

Stress is a quality that is directly proportional to the force acting on an object and inversely proportional to its cross sectional area.

Mathematically,

$$\text{Stress} = \frac{F}{A} \quad 4$$

1.3.2 STRAIN

Strain is measure of the degree of deformation.

Mathematically,

$$\text{Strain} = \frac{\text{extension}}{\text{original length}} = \frac{e}{l_0} \quad 5$$

It is found that, for sufficiently small stress, strain is proportional to stress. The constant of proportionality depends on the material being deformed and on the nature of deformation. We call this proportionality constant the **ELASTIC MODULES (E.M)**. The E.M. is the ratio of the stress to the resulting strain.

Mathematically,

$$E.M = \frac{\text{Stress}}{\text{Strain}} \quad 6$$

In a very real sense it is a comparison of what is done to a solid object (a force is applied) and how that object responds (it deforms to some extent).

1.4 Young's Modulus: Elasticity in Length

Consider a long bar of cross-sectional area A and initial length L_i that is clamped at one end. When an external force is applied perpendicular to the cross-sectional area, internal forces in the bar resist distortion (stretching), but the bar attains an equilibrium in which its final length L_f is greater than L_i and the external force is exactly balanced by internal forces. In such a situation, the bar is said to be stressed.

Therefore, we define the **tensile stress** as the ratio of the magnitude of the external force F to the cross-sectional area A . The **tensile strain** in this case is defined as the ratio of the change in length ΔL to the original length L_i

So, we define young's modulus by a combination of these two ratios:

$$Y = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{F/A}{\Delta L/L_i} \quad 7$$

Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression. Its S.I. unit N/m^2 .

1.5 Shear modulus: Elasticity of shape

Another type of deformation occurs when an object is subjected to a force tangential to one of its faces while the opposite face is held fixed by another force. If the object is originally a rectangular block, a shear stress results in a shape whose cross-section is a parallelogram.

Therefore, **shear stress** is defined as the ratio of the tangential force to the area A of the face being sheared.

The **shear strain** is defined as the ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and h is the height of the object.

In terms of these quantities, the shear modulus is

$$S = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{F/A}{\Delta x/h} \quad 8$$

1.6 Bulk Modulus: Volume Elasticity

Bulk modulus characterizes the response of a substance to uniform squeezing or to a reduction in pressure when the object is placed in a partial vacuum. An object subjected to this type of deformation undergoes a change in volume but no change in shape.

The **volume stress** is defined as the ratio of the magnitude of the normal force F to the area A . The quantity $P = F/A$ is called the pressure. Now, if the pressure in an object changes by an amount $\Delta P = \Delta F/A$, then the object will experience a volume change ΔV .

The **volume strain** is equal to the change in volume ΔV divided by the initial volume V_i .

Thus, from equation (3), we can characterize a volume (bulk) compression in terms of a bulk modulus, which is defined as

$$B = \frac{\text{Volume Stress}}{\text{Volume Strain}} = \frac{-\Delta F/A}{\Delta V/V_i} \quad 9$$

Assignment 1

1. Support a new flat eraser (with gum) on two parallel pencils at least 3cm apart. Press down on the middle of the top surface just enough to make the top surface of the eraser curve a bit.
 - i. Is the top face under tension or compression?
 - ii. How about the bottom?
2. Why does a flat slab of concrete supported at the end crack on the bottom face and not the top?

2.0 FLUID MECHANICS

A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces and by forces exerted by walls of a container. E.g. Liquids and gases.

2.1 PRESSURE:

Fluids do not sustain shearing stresses or tensile stresses; the only stress that can be exerted on an object submerged in a fluid is one that tends to compress the object. In other words, the force exerted by a fluid on an object is always perpendicular to the surfaces of the object.

Mathematically,

$$P = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} = Nm^{-2}$$

The S.I unit is Nm^{-2} or *Pascal*

$1pa = 1Nm^2$ & $1bar = 10^5 N/m^2 = 10^5 pa$.

It is a scalar quantity.

Example 1:

Snowshoes keep you from sinking into soft snow because they spread the downward force you exert on the snow over a large area reducing the pressure on the snow's surface.

Example 2:

The mattress of a waterbed is 2.00m long by 2.00m wide and 30.0cm deep.

- Find the weight of the water in the mattress
- Find the pressure exerted by the water on the floor when the bed rests in its normal position. Assume that the entire lower surface of the bed makes contact with the floor.

Solution:

(a) $h = 30/100 = 0.3m$

Density of H_2O is $1000kg/m^3$

$$\therefore \text{The mass, } M = \rho \times v = 1000 \times 1.20 = 1.20 \times 10^3 kg$$

$$\therefore \text{The weight of the water; } w = mg = (1.20 \times 10^3) \times 9.80 = 1.18 \times 10^4 N$$

(b) When the bed is in its normal position, the cross-sectional area is 4.00m^2 ;

$$P = \frac{F}{A} = \frac{1.18 \times 10^4 \text{ N}}{4.00\text{m}^2} = 2.95 \times 10^3 \text{ Pa}$$